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Towards Complete Neutrino Mixing Matrix and CP-Violation

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The compelling experimental evidences for oscillations of solar, atmospheric and reactor neutrinos imply the existence of 3-neutrino mixing in vacuum. We review the phenomenology of 3-neutrino mixing, and the current data on the 3-neutrino mixing parameters. The opened questions and the main goals of future research in the field of neutrino mixing and oscillations are outlined. A phenomenological approach for understanding the pattern of neutrino mixing as an interplay between the mixing, arising from the charged lepton sector, and bimaximal mixing, arising from a neutrino Majorana mass matrix, is considered with emphasis on the $CP-$violating case. We comment also on planned future steps in the experimental studies of ν-mixing.

1. Introduction

The hypothesis of neutrino oscillations was formulated in [1]. In [2] it was suggested that the solar ν_e can take part in oscillations involving another active or sterile neutrino. The evidences of solar neutrino ($\nu_{\odot}$-) oscillations obtained first in the Homestake experiment and strengthened by the results of Kamioaknde, SAGE and GALLEX/GNO experiments [3,4], were made compelling in the last several years by the data of Super-Kamiokande (SK), SNO and KamLAND (KL) experiments [5,6,7]. Under the plausible assumption of CPT-invariance, the results of the KL reactor neutrino experiment [7] established the large mixing angle (LMA) MSW oscillations/transitions [8] as the dominant mechanism at the origin of the observed solar ν_e deficit. The Kamiokande experiment [4] provided the first evidences for oscillations of atmospheric ν_μ and $\bar{\nu}_\mu$, while the data of the Super-Kamiokande experiment made the case of atmospheric neutrino oscillations convincing [9]. Evidences for oscillations of neutrinos were obtained also in the first long baseline accelerator neutrino experiment K2K [10]. Indications for ν-oscillations were reported by the LSND collaboration [11].

The recent new SK data on the L/E-dependence of multi-GeV μ-like atmospheric neutrino events [9], L and E being the distance traveled by neutrinos and the ν energy, and the new spectrum data of KL and K2K experiments [12,13], presented at this Conference, are the latest significant contributions to the remarkable progress made in the last several years in the studies of ν-oscillations. For the first time the data exhibit directly the effects of the oscillatory dependence on L/E and E of the probabilities of ν-oscillations in vacuum [14]. We begin to "see" the oscillations of neutrinos. As a result of these magnificent developments, the oscillations of solar ν_e, atmospheric ν_μ and $\bar{\nu}_\mu$, accelerator ν_μ (at $L \sim$250 km) and reactor $\bar{\nu}_e$ (at L ~180 km), driven by nonzero ν-masses and ν-mixing, can be considered as practically established.

2. The Neutrino Mixing Parameters

The SK atmospheric neutrino and K2K data are best described in terms of dominant 2-neutrino $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) vacuum oscillations. The best fit values and the 99.73% C.L. allowed ranges of the atmospheric neutrino ($\nu_{\rm A}$-) oscillation parameters read [9]:
$|\Delta m^2_{\rm A}|$=2.1$\times$10^{-3} eV^2, $\sin^2 2\theta_{\rm A} = 1.0$,
$|\Delta m^2_{\rm A}|$=(1.3 – 4.2)$\times$10^{-3} eV^2, $\sin^2 2\theta_{\rm A} \geq 0.85$.
The sign of $\Delta m^2_{\rm A}$ and of $\cos 2\theta_{\rm A}$, if $\sin^2 2\theta_{\rm A} \neq 1.0$, cannot be determined using the existing data. The latter implies that when, e.g., $\sin^2 2\theta_{\rm A} = 0.92$, one has $\sin^2 \theta_{\rm A} \cong 0.64$ or 0.36.

The combined 2-neutrino oscillation analysis of the solar neutrino and the new KL 766.3 Ty spec-

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trum data shows [12,15] that the $\nu_\odot$-oscillation parameters lie in the low-LMA region :

$\Delta m^2_\odot = (7.9^{+0.6}_{-0.5}) \times 10^{-5}\ \mathrm{eV}^2$, $\tan^2\theta_\odot = (0.40^{+0.09}_{-0.07})$. The value of $\Delta m^2_\odot$ is determined with a remarkably high precision. The high-LMA solution is excluded at more than 3σ. Maximal $\nu_\odot$-mixing is ruled out at $\sim 6\sigma$; at 95% C.L., $\cos 2\theta_\odot \geq 0.27$. One also has: $\Delta m^2_\odot / |\Delta m^2_{\mathrm{A}}| \sim 0.04 \ll 1$.

The interpretation of the solar and atmospheric neutrino, and of K2K and KL data in terms of ν-oscillations requires the existence of 3-ν mixing in the weak charged lepton current:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}, \quad l = e, \mu, \tau,$$

where $\nu_{j\mathrm{L}}$ is the field of neutrino ν_j having a mass m_j and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) ν-mixing matrix [1,16]. All existing ν-oscillation data, except the data of LSND experiment [3] [11], can be described assuming 3-ν mixing in vacuum and we will consider only this possibility. The minimal 4-ν mixing scheme which could incorporate the LSND indications for ν-oscillations is strongly disfavored by the data [18]. The ν-oscillation explanation of the LSND results is possible assuming 5-ν mixing [19].

The PMNS matrix can be parametrized by 3 angles and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles, by 1 or 3 CP-violation (CPV) phases [20,21]. In the standardly used parameterization (see, e.g., [22]),

$$U_{\mathrm{PMNS}} = V(\theta_{12}, \theta_{13}, \theta_{23}, \delta)\ \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}),$$

where $V(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ is a CKM-like matrix, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CPV phase and α, β are two Majorana CPV phases [20,21]. One can identify $\Delta m^2_\odot = \Delta m^2_{21} > 0$. In this case $|\Delta m^2_{\mathrm{A}}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$, $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_{\mathrm{A}}$. The angle θ_{13} is limited by the data from the CHOOZ and Palo Verde experiments [23]. The limit depends strongly on $|\Delta m^2_{\mathrm{A}}|$ (see, e.g, [24]). The existing ν_{A}-data is essentially insensitive to θ_{13} obeying the CHOOZ limit [9]. The probabilities of survival of reactor $\bar{\nu}_e$ and solar ν_e, relevant for the interpretation of the KL, CHOOZ and $\nu_\odot$- data, depend on θ_{13}:

$$P^{3\nu}_{\mathrm{KL}} \cong \sin^4\theta_{13} + \cos^4\theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21}\mathrm{L}}{4E}\right],$$

$$P^{3\nu}_{\mathrm{CHOOZ}} \cong 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31}\mathrm{L}}{4\mathrm{E}},$$

$$P^{3\nu}_\odot \cong \sin^4\theta_{13} + \cos^4\theta_{13}\ P^{2\nu}_\odot(\Delta m^2_{21}, \theta_{12}; \theta_{13}),$$

where $P^{2\nu}_\odot$ is the 2-ν mixing solar ν_e survival probability [25,26,27,28,29] in the case of transitions driven by Δm^2_{21} and θ_{12}, in which the solar e^- number density N_e is replaced by $N_e \cos^2\theta_{13}$ [30],

$$P^{2\nu}_\odot = \bar{P}^{2\nu}_\odot + P^{2\nu}_{\odot\ \mathrm{osc}},$$

$$\bar{P}^{2\nu}_\odot = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta^m_{12}(t_0) \cos 2\theta_{12},$$

$$P' = \frac{\exp(-2\pi r_0 \frac{\Delta m^2_{21}}{2E} \sin^2\theta_{12}) - \exp(-2\pi r_0 \frac{\Delta m^2_{21}}{2E})}{1 - \exp(-2\pi r_0 \frac{\Delta m^2_{21}}{2E})} \qquad (1)$$

Here $\bar{P}^{2\nu}_\odot$ is the average probability [31,25,28], $P^{2\nu}_{\odot\ \mathrm{osc}}$ is an oscillating term [25,26,28,29], P' is the "double exponential" jump probability [25] and r_0 is the "running" scale-height of the change of N_e along the ν-trajectory in the Sun [4] [25,27,29]. In the LMA solution region one has [26] $P^{2\nu}_{\odot\ \mathrm{osc}} \cong 0$. Using the 3σ allowed range of $|\Delta m^2_{\mathrm{A}}| = |\Delta m^2_{31}|$ [9] and performing a combined analysis of the solar neutrino, CHOOZ and KL data, one finds [15]:

$$\sin^2\theta_{13} < 0.05, \qquad 99.73\%\ \mathrm{C.L.}$$

Similar constraint is obtained from a global 3-ν

[3] In the LSND experiment indications for oscillations $\bar{\nu}_\mu \to \bar{\nu}_e$ with $(\Delta m^2)_{\mathrm{LSND}} \simeq 1\ \mathrm{eV}^2$ were obtained. The LSND results are being tested in the MiniBooNE experiment [17].

[4] The claims in [32] that in the LMA region "The double exponential formula is not valid... It requires production of neutrinos far above the resonance region in the density scale. This formula is not applicable in the range $\Delta m^2 \cos 2\theta/(2E) \sim (1.6 - 8.0) \times 10^{-6}\ \mathrm{eV}^2/\mathrm{MeV}$ for which the density in the production point turns out to be close to the resonance density." are incorrect and/or misleading. The analyses and the extensive numerical studies performed in [27,33,29] show that expression (1) for $\bar{P}^{2\nu}_\odot$ provides a high precision description of the average solar ν_e survival probability in the Sun for any values of Δm^2_{21} and θ_{12} (the relevant error does not exceed $\sim$(2-3)%), including the values from the LMA region. Actually, it follows from the results in [32] that the use of the double exponential expression for P' [25], eq. (1), for description of the LMA transitions brings in an imprecision in $\bar{P}^{2\nu}_\odot$ which does not exceed $\sim 10^{-6}$. Similarly, the claim in [32] that "... for the LMA solution, when the final mixing angle is large, one cannot use the Landau-Zener probability as an approximation for P_c. ", P_c being the jump probability, is incorrect, see [34,27,33].

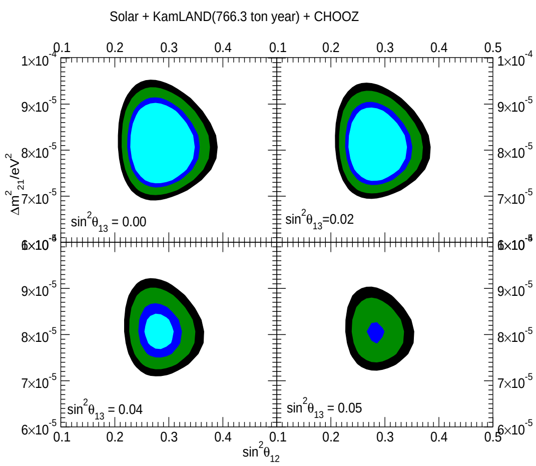

Figure 1. The 90%, 95%, 99% and 99.73% C.L. allowed regions in the Δm^2_{21} -$\sin^2\theta_{12}$ plane, obtained in a 3-ν oscillation analysis of the solar neutrino, KL and CHOOZ data [15].

oscillation analysis of the data [18,35]. In Fig. (1) we show the allowed regions in the Δm^2_{21} $-$ $\sin^2\theta_{12}$ plane for few fixed values of $\sin^2\theta_{13}$ [15], obtained using, in particular, eq. (1) for $\bar{P}^{2\nu}_{\odot}$.

Thus, the fundamental parameters characterizing the 3-neutrino mixing are:
• the 3 angles θ_{12}, θ_{23}, θ_{13},
• depending on the nature of ν_j - 1 Dirac (δ), or 1 Dirac + 2 Majorana (δ, α, β), CPV phases, and
• the 3 neutrino masses, $m_1,\ m_2,\ m_3$.

It is convenient to express the two larger masses in terms of the third mass and the measured $\Delta m^2_{\odot}$ =Δm^2_{21} >0 and $\Delta m^2_{\rm A}$. In the convention we are using, the two possible signs of $\Delta m^2_{\rm A}$ correspond to two types of ν-mass spectrum:
• with normal hierarchy, $m_1 < m_2 < m_3$, $\Delta m^2_{\rm A}$=$\Delta m^2_{31} > 0$, $m_{2(3)}$=$(m_1^2 + \Delta m^2_{21(31)})^{\frac{1}{2}}$,and
• with inverted hierarchy, $m_3 < m_1 < m_2$, $\Delta m^2_{\rm A}$=Δm^2_{32} <0, m_2=$(m_3^2 - \Delta m^2_{32})^{\frac{1}{2}}$, etc.
The spectrum can also be
• *normal hierarchical (NH)*: $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m^2_{\odot})^{\frac{1}{2}}$ ~0.009 eV, $m_3 \cong |\Delta m^2_{\rm A}|^{\frac{1}{2}}$ ~0.045; or
• *inverted hierarchical (IH)*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m^2_{\rm A}|^{\frac{1}{2}}$ ~0.045 eV; or
• *quasi-degenerate (QD)*: $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m^2_{\rm A}|$, $m_0 \gtrsim 0.20$ eV.

After the spectacular experimental progress made in the studies of ν-oscillations, further understanding of the structure of the ν-masses and ν-mixing, of their origins and of the status of the CP-symmetry in the lepton sector requires a large and challenging program of research to be pursued in neutrino physics. The main goals of this research program should include:
• High precision measurement of the solar and atmospheric neutrino oscillations parameters, Δm^2_{21}, θ_{21}, and Δm^2_{31}, θ_{23}.
• Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, θ_{13} - the only small mixing angle in $U_{\rm PMNS}$.
• Determination of the $sign(\Delta m^2_{31})$ and of the type of ν-mass spectrum (NH, IH, QD, etc.).
• Determining or obtaining significant constraints on the absolute scale of ν-masses, or on $min(m_j)$.
• Determining the nature–Dirac or Majorana, of massive neutrinos ν_j.
• Establishing whether the CP-symmetry is violated in the lepton sector a) due to the Dirac phase δ, and/or b) due to the Majorana phases α and β if ν_j are Majorana particles.
• Searching with increased sensitivity for possible manifestations, other than flavour neutrino oscillations, of the non-conservation of the individual lepton charges L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.
• Understanding at fundamental level the mechanism giving rise to neutrino masses and mixing and to L_l-non-conservation, i.e., finding the *Theory of neutrino mixing*. This includes understanding the origin of the patterns of ν-mixing and ν-masses suggested by the data. Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of new fundamental symmetry of particle interactions? Is there any relations between quark mixing and neutrino mixing, e.g., does the relation $\theta_{12} + \theta_c$=$\pi/4$, where θ_c is the Cabibbo angle, hold? Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$? What is the physical origin of CPV phases in $U_{\rm PMNS}$? Is there any relation (correlation) between the (values of) CPV phases and mixing angles in $U_{\rm PMNS}$? Progress in the theory of ν-mixing might also lead, in particular, to a better understanding of the mechanism of gen-

eration of baryon asymmetry of the Universe [36].

Obviously, the successful realization of the experimental part of this research program would be a formidable task and would require many years.

The mixing angles, θ_{21}, θ_{23} and θ_{13}, Dirac CPV phase δ and Δm^2_{21} and Δm^2_{31} can, in principle, be measured with a sufficiently high precision in a variety of ν-oscillation experiments (see further). These experiments, however, cannot provide information on the absolute scale of ν- masses and on the nature of massive neutrinos ν_j. The flavour neutrino oscillations are insensitive to the Majorana CPV phases α and β [20,37]. Establishing whether ν_j have distinct antiparticles (Dirac fermions) or not (Majorana fermions) is of fundamental importance for understanding the underlying symmetries of particle interactions [38] and the origin of ν-masses. If ν_j are Majorana fermions, getting experimental information about the Majorana CPV phases in $U_{\rm PMNS}$ is a remarkably challenging problem. [39,40,41]. The phases α and β can affect significantly the predictions for the rates of the (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. in a large class of supersymmetric theories with see-saw mechanism of neutrino mass generation (see, e.g., [42]). Majorana CPV phases might be at the origin of the baryon asymmetry of the Universe [36].

3. The Pattern of Neutrino Mixing

The ν-oscillation data suggest that $\theta_{12} \cong \pi/6$, $\theta_{23} \cong \pi/4$, and $\theta_{13} \equiv \epsilon < \pi/12$. Thus, the PMNS matrix is very different from the CKM matrix:

$$U_{\rm PMNS} \cong \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (2)$$

where the CPV phases have been suppressed. Understanding the origin of the emerged patterns of ν-mixing and of $\Delta m^2_{21,31}$, is one of the central problems in today's neutrino physics.

$U_{\rm PMNS}$ is close in form to a bimaximal mixing matrix, for which θ_{12}=θ_{23}=$\pi/4$ and θ_{13}=0:

$$U_{\rm bimax} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (3)$$

Whereas the data favor $\theta_{23} = \pi/4$ and allows for $\theta_{13} = 0$, $\theta_{12} = \pi/4$ is ruled out at $\sim 6\sigma$ [12,15]. The deviation of θ_{12} from $\pi/4$ can be parameterized with a parameter λ, which is very similar in value to the Cabibbo angle [43]:

$$\sin\theta_{21} \cong \frac{1}{\sqrt{2}}(1-\lambda),\ \ \lambda \cong (0.20 - 0.25) \sim \theta_C.$$

The implied relation $\theta_\odot = \pi/4 - \theta_C$, if confirmed experimentally, might be linked to GUT's [44].

It is natural to suppose that [45,46] [5]

$$U_{\rm PMNS} = U^\dagger_{\rm L}(\lambda)\ U_\nu,\ \ U_\nu = U_{\rm bimax},$$

where $U^\dagger_{\rm L}(\lambda)$ arises from diagonalization of the charged lepton mass matrix and U_ν diagonalizes a neutrino Majorana mass matrix M_ν. The inequality $\Delta m^2_{21} \ll |\Delta m^2_{31}|$ and the form of $U_{\rm bimax}$ can be associated with an *approximate* symmetry of M_ν, implying (for $U_{\rm L}$=$\mathbf{1}$) the conservation of the lepton charge [45]:

$$L' = L_e - L_\mu - L_\tau \qquad (4)$$

If θ^l_{ij} are the 3 angles of CKM-like parametrization of $U_{\rm L}(\lambda)$ and $\sin\theta^l_{ij} \equiv \lambda_{ij}$, 3 generic cases are compatible with the ν-mixing data [46]:

- all λ_{ij} small, $\lambda_{ij} \lesssim 0.35$;
- $\lambda_{23} = 1$, $\lambda_{12}, \lambda_{13} \lesssim 0.35$;
- all λ_{ij} large, e.g., $\lambda_{ij} \geq \frac{1}{\sqrt{2}}$.

For $\lambda_{ij} \lesssim 0.35$, the data imply: $\lambda_{23} \lesssim 0.19$, $\lambda_{12} \gg \lambda_{13}$ (for $\lambda_{12,13} > 0$), λ_{12} $\cong$(0.21-0.25), $\lambda_{13} \lesssim 0.03$. With all CPV phases set to 0, θ_{ij} are expressed in terms of λ_{kl} and their values are correlated. The deviation of θ_{12} from $\pi/4$ is determined by s_{13}=$\sin\theta_{13}$, which typically implies $s^2_{13} \gtrsim 0.01$. Consider two simple cases [46].

- Hierarchy of λ_{ij}: $\lambda_{12} \equiv \lambda$, $\lambda_{23} \sim \lambda^2$, $\lambda_{13} \sim \lambda^3$. Then $s_{13} \cong \lambda/\sqrt{2}$, $\sin^2 2\theta_{23}$=1-16s^4_{13}, $\tan^2\theta_{12}$=1-4s_{13}(1-2s_{13}+4s^2_{13}), and $\sin^2 2\theta_{23} \gtrsim 0.96$; we also have $s^2_{13} \gtrsim 0.01$ for $\tan^2\theta_{12} \leq 0.58$ [15].
- $\lambda_{12} \equiv \lambda$, $\lambda_{23} \sim \lambda/2$ (mild hierarchy), $\lambda_{13} \sim \lambda^3$. Now $\sin^2 2\theta_{23} \gtrsim 0.90$.

If λ_{23}=1, $\lambda_{12(13)} \lesssim 0.35$, one has $\sin^2 2\theta_{23} \gtrsim 0.997$, while for $\lambda_{ij} \geq \frac{1}{\sqrt{2}}$, $\sin^2 2\theta_{23} \gtrsim 0.95$ [46]. Obviously, a sufficiently precise measurement of $\sin^2 2\theta_{23}$ would allow to distinguish between the three possibilities.

If CP is not conserved, we have [47]

$$U_{\rm PMNS} = U^\dagger_{\rm L} U_\nu = \tilde{U}^\dagger_{\rm lep} P_\nu \tilde{U}_\nu Q_\nu,$$

[5] Extensive list of references on the subject is given in [46].

where, in general, $\tilde{U}_{\rm lep}$, $\tilde{U}_\nu$ are CKM-like matrices each containing 3 angles and 1 CPV phase, $P_\nu = {\rm diag}(1, e^{i\phi}, e^{i\omega})$ and $Q_\nu = {\rm diag}(1, e^{i\rho}, e^{i\sigma})$. Suppose that $\tilde{U}_\nu = U_{\rm bimax}$ and arises from diagonalization of the simplest possible M_ν [46],

$$M_\nu = \frac{m}{\sqrt{2}} \begin{pmatrix} 0 & e^{-i\alpha'} & e^{-i\beta'} \\ e^{-i\alpha'} & 0 & \epsilon\, e^{-i\gamma'} \\ e^{-i\beta'} & \epsilon\, e^{-i\gamma'} & 0 \end{pmatrix}$$

Here α', β', γ' are phases, $m^2 \cong -\Delta m^2_{\rm A}$=$\Delta m^2_{23}$ and $\Delta m^2_\odot$ =$\Delta m^2_{21} \cong \sqrt{2}\epsilon m^2$, $\epsilon \sim 0.03$. The ν-mass spectrum is IH. In the limit $\epsilon = 0, \tilde{U}_{\rm lep} = \mathbf{1}$, $L' = L_e - L_\mu - L_\tau$ is conserved [45]. For $\tilde{U}_{\rm lep} \neq \mathbf{1}$, $(\alpha' - \gamma')$=ω and $(\beta' - \gamma')$=ϕ are physical CPV phases, $Q_\nu = \mathbf{1}$, and $U_{\rm PMNS}$=$\tilde{U}^\dagger_{\rm lep} P_\nu U_{\rm bimax}$. The Dirac and Majorana phases in $U_{\rm PMNS}$ have the same "source" - the CPV phases in $\tilde{U}^\dagger_{\rm lep}$ and P_ν, ψ and ϕ, ω. Even θ_{ij} depend on the latter.

For, e.g., "small" λ_{12}=λ, λ_{23}=$A\lambda$, λ_{13}=$B\lambda^3$, $A, B \sim 1$, one finds up to terms $\mathcal{O}(\lambda^3)$ [46]:
$\tan^2\theta_{12} \cong 1 - 2\sqrt{2}\lambda c_\phi + 2(2c^2_\phi - \sqrt{2}Ac_\omega)\lambda^2$,
$\sqrt{2}s_{13} \cong \lambda$–$A\lambda^2 c_{\omega-\phi}$, $\sin^2 2\theta_{23} \cong$1–$4A^2\lambda^2 c_{\omega-\phi}$,
where c_ϕ=$\cos\phi$, etc.
The rephasing invariants associated with CPV phases δ, β and α-β read, respectively: $J_{CP} = {\rm Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\}$ [48,49],
S_1=Im$\{U_{e1}U^*_{e3}\}$, S_2=Im$\{U_{e2}U^*_{e3}\}$ [50].
J_{CP} controls the magnitude of CP and T violating effects in ν-oscillations [49]; S_1, S_2 appear in the effective Majorana mass $|<\!m\!>|$ in $(\beta\beta)_{0\nu}$-decay [22]. In general, J_{CP}, S_1 and S_2 are independent. However, in the scheme with approximate $L_e - L_\mu - L_\tau$ symmetry we are considering, and to leading order in λ we find [46]:

$$J_{CP} \cong \frac{S_1}{2\sqrt{2}} \cong \frac{S_2}{2\sqrt{2}}.$$

Thus, *the magnitude of CP–violating effects in ν-oscillations is directly related to the magnitude of CP–violating effects associated with the Majorana nature of neutrinos.* One also finds [46]:

$$|<\!m\!>| \cong \sqrt{|\Delta m^2_{\rm A}|}\ \ |\cos 2\theta_\odot + i\ 8 J_{CP}|\ ,$$

i.e., J_{CP} determines the deviation of $|<\!m\!>|$ from its minimal value (for IH spectrum) [22].

The approach to understand the pattern of ν-mixing discussed above is by no means unique (see, e.g., [51,52]). It demonstrates that Dirac and Majorana CPV phases (and effects) can be related, and that θ_{ij} can depend on CPV phases.

4. Comments on Future Progress

Future progress in the studies of ν-mixing will be crucial for understanding at fundamental level the mechanism generating it. The requisite data is forseen to be provided by
- $\nu_\odot-$ and $\nu_{\rm A}-$ experiments:
SK, SNO, SAGE, BOREXINO [53], LowNu [54], and SK ($\nu_{\rm A}$), MINOS ($\nu_{\rm A}$) [55], INO ($\nu_{\rm A}$) [56];
- Reactor $\bar{\nu}_e$ experiments with $L \sim$(1 - 180) km;
- Accelerator experiments:
K2K (L ~250 km), MINOS (L ~730 km), OPERA and ICARUS [57,58] (L ~730 km);
- Experiments with super beams:
T2K (L ~295 km) [59], NOνA (L ~800 km) [60] SPL+$\beta-$beams with UNO (1 megaton) detector (CERN-Frejus, L ~135 km) [61];
- ν-Factory experiments (L ~3000;7000 km) [62];
- $(\beta\beta)_{0\nu}-$ and ^{3}H $\beta-$ decay experiments [63,64];
- Astrophysical/cosmological observations [65].

Absolute Neutrino Mass Measurements.
The Troitzk and Mainz ^{3}H β-decay experiments provided information on the $\bar{\nu}_e$ mass [64]: $m_{\bar{\nu}_e}$ <2.2 eV at 95% C.L. The KATRIN experiment [64] expected to start in 2007, is planned to reach sensitivity to $m_{\bar{\nu}_e}$ ~0.20 eV (95% C.L.), and thus to probe the region of QD ν-mass spectrum. In this region $m_{1,2,3} \cong m_0 \cong m_{\bar{\nu}_e}$.

The CMB data of the WMAP experiment were used to obtain an upper limit on $\sum_j m_j$ [65]:
$\sum_j m_j <$ (0.7–1.8) eV (95% C.L.).
The WMAP and future PLANCK experiments can be sensitive to $\sum_j m_j \cong$ 0.40 eV. Data on weak lensing of galaxies by large scale structure, combined with WMAP and PLANCK data, may allow one to determine $\sum_j m_j$ with an uncertainty of ~(0.04–0.10) eV (see, e.g., [65]).

$(\beta\beta)_{0\nu}$**–Decay Experiments**. The $(\beta\beta)_{0\nu}$-decay experiments [63] have a remarkable physics potential [6]. They can establish the Majorana nature of neutrinos ν_j. If ν_j are Majorana particles, they can provide unique information
- on the type of ν-mass spectrum [67,22,68],

[6] For detailed discussion and list of references see, e.g., [66]

• on the absolute scale of ν-masses [69], and
• on the Majorana CPV phases in $U_{\rm PMNS}$ [39,41].
The $(\beta\beta)_{0\nu}$-decay, (A,Z)→(A,Z+2)+e^-+e^-, is allowed if ν_j are Majorana particles (see, e.g., [38]). The nature - Dirac or Majorana, of neutrinos ν_j is related to the fundamental symmetries of particle interactions. Neutrinos ν_j will be Dirac fermions if particle interactions conserve the total lepton charge L. They can be Majorana particles if there does not exist any conserved lepton charge. Massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of ν-mass generation (see, e.g., [70]), which also provides an attractive explanation of the smallness of ν-masses and - through leptogenesis theory (see [36]), of the baryon asymmetry of the Universe.

If ν_j are Majorana fermions, the $(\beta\beta)_{0\nu}$-decay amplitude of interest has the form (see, e.g., [22]): $A(\beta\beta)_{0\nu} \cong \langle m \rangle\, M$, where M is the corresponding nuclear matrix element (NME) and

$$|{<}\,m\,{>}| = \left|m_1|U_{\rm e1}|^2 + m_2|U_{\rm e2}|^2 e^{i\alpha} + m_3|U_{\rm e3}|^2 e^{i\beta}\right|$$

$|U_{\rm e1}|$=$c_{12}c_{13}$, $|U_{\rm e2}|$=$s_{12}c_{13}$, $|U_{\rm e3}|$=s_{13}. In the case of CP-invariance one has [71],
$\eta_{21} \equiv e^{i\alpha}$=±1, $\eta_{31} \equiv e^{i\beta}$=±1, $\eta_{21(31)}$ being the relative CP-parity of Majorana neutrinos $\nu_{2(3)}$ and ν_1. Thus, $|{<}\,m\,{>}|$ depends [72] on $\theta_\odot$, θ_{13}, $\Delta m^2_{\rm A}$, $\Delta m^2_\odot$ as well as on $min(m_j)$, Majorana phases α, β and the ν-mass spectrum. The predicted value of $|{<}\,m\,{>}|$ for $\sin^2\theta_{13}$=0.04 and 90% C.L. allowed values of $\Delta m^2_{\rm A}$ and $\Delta m^2_\odot$, $\theta_\odot$ (Fig. 1) is shown as function of $min(m_j)$ in Fig. 2.

The main features of the predictions for $|{<}\,m\,{>}|$ are [68,69,66]: i) for NH spectrum, typically $|{<}\,m\,{>}| \lesssim 0.006$ eV, and $|{<}\,m\,{>}| \sim 0$ is possible; ii) for IH spectrum, $|{<}\,m\,{>}| \gtrsim \sqrt{|\Delta m^2_{\rm A}|}\cos 2\theta_\odot \gtrsim 0.012$ eV and $|{<}\,m\,{>}| \lesssim \sqrt{|\Delta m^2_{\rm A}|} \lesssim 0.06$ eV; iii) in the case QD spectrum, $|{<}\,m\,{>}| \gtrsim m_0\cos 2\theta_\odot \gtrsim 0.05$ eV and $|{<}\,m\,{>}| \lesssim m_0$, with $m_0 \gtrsim 0.2$ eV and $m_0 < 2.2$ eV [64], $m_0 < 0.6$ eV [65]. Thus, for IH and QD spectra, $|{<}\,m\,{>}|$ is limited from below.

Many experiments have searched for $(\beta\beta)_{0\nu}$-decay [74]. The best sensitivity was achieved in Heidelberg-Moscow ^{76}Ge experiment [75]. A positive signal at $>3\sigma$, corresponding to $|{<}\,m\,{>}| = (0.1 - 0.9)$ eV at 99.73% C.L., is claimed to be observed [75]. Two experiments,

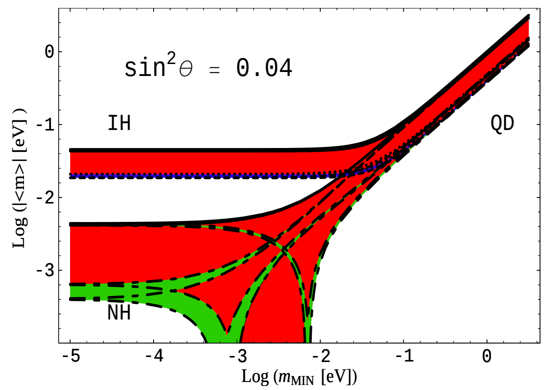

Figure 2. The value of $|{<}\,m\,{>}|$ as function of $min(m_j)$ for $\sin^2\theta_{13}$=0.04 and 90% C.L. allowed ranges of values of $\Delta m^2_{\rm A}$, $\Delta m^2_\odot$ and $\theta_\odot$ [68,73].

NEMO3 (with ^{100}Mo and ^{82}Se) [76] and CUORICINO (with ^{130}Te) [77], designed to reach sensitivity to $|{<}\,m\,{>}| \sim$(0.2-0.3) eV, announced first results: $|{<}\,m\,{>}| <$ (0.7–1.2) eV [76] and $|{<}\,m\,{>}| <$ (0.3 – 1.6) eV [77] (90% C.L.), where estimated uncertainties in the NME [78] are accounted for. A number of projects aim at sensitivity to $|{<}\,m\,{>}| \sim$(0.01–0.05) eV [63]: CUORE (^{130}Te), GENIUS (^{76}Ge), EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), XMASS (^{136}Xe), etc. These experiments will probe the region corresponding to IH and QD spectra and test the positive result claimed in [75]. The knowledge of the relevant NME with sufficiently small uncertainty is crucial for obtaining quantitative information on the ν-mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life. *In view of their importance for understanding the origin of ν- masses and mixing, performing few $(\beta\beta)_{0\nu}$-decay experiments with sensitivity to $|{<}\,m\,{>}| \sim$(0.01–0.05) eV (or better) should have highest priority in the future studies of ν-mixing.*

The CHOOZ Angle θ_{13}. The angle θ_{13} plays extremely important role in the phenomenology of ν-oscillations. It controls together with $\sin\delta$ the magnitude of CP- and T-violating effects in $\nu-$oscillations. It controls the sub-dominant $\nu_\mu \leftrightarrow \nu_e$ and $\bar\nu_\mu \leftrightarrow \bar\nu_e$ oscillations of $\nu_{\rm A}$ and in the accelerator experiments MINOS, OPERA, ICARUS, T2K, NOνA, etc. and at $\nu-$factories.

The value of $|{<}m{>}|$ in $(\beta\beta)_{0\nu}$-decay in the case of NH spectrum depends on $\sin^2\theta_{13}$ [22,69]. The knowledge of θ_{13} is crucial for finding the correct theory of ν-mixing as well.

If $\sin^2 2\theta_{13} \lesssim (0.01 - 0.008)$, the CPV effects in ν-oscillations would be too small to be observed in T2K and NOνA experiments [79,80]. Thus, the future program of searches for CPV effects in accelerator experiments depends critically on the value of θ_{13}. The sensitivity of future experiments with *conventional beams (MINOS+ICARUS+OPERA)*, with off-axis super beams, T2K-SK, NOνA (NuMI), and with reactor $\bar{\nu}_e$ - Double-CHOOZ [81], to $\sin^2 2\theta_{13}$, as function of $|\Delta m^2_{31}|$, is illustrated in Fig. 3.

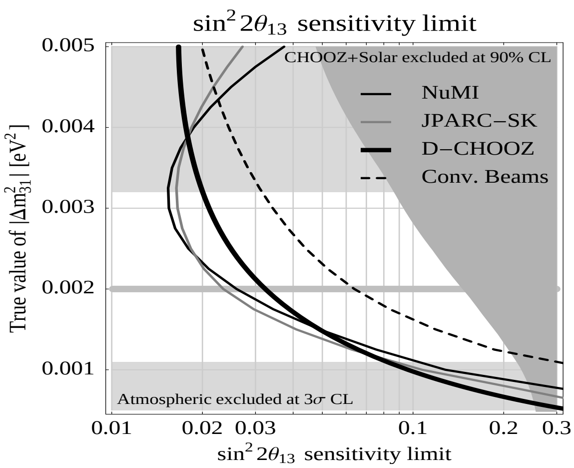

Figure 3. The sensitivity of future experiments to $\sin^2 2\theta_{13}$ [79].

There are several proposals for reactor $\bar{\nu}_e$ experiments with baseline L ~(1-2) km [82,83], which could imporve the current limit $\sin^2\theta_{13}$ <0.05 by a factor of (5-10) [81]. The most advanced in preparation is the Double-CHOOZ project. *The reactor θ_{13} experiments, in our view, should have highest priority in the program of research in ν-physics: they can compete in sensitivity with accelerator experiments (T2K-SK, NOνA) and can be done on relatively short (for experiments in this field) time scale. The planning of experiments to study CPV effects in ν-oscillations would benefit significantly from the results of a high precision reactor θ_{13} experiment.*

Measuring $\Delta m^2_\odot \equiv \Delta m^2_{21}$ and $\theta_\odot \equiv \theta_{12}$. The current solar neutrino and KL 766.3 Ty data determine Δm^2_{21} and $\sin^2\theta_{12}$ with uncertainties of 12% and 24% at 3σ [84]. Accounting for possible reduction of errors in the data from the phase-III of SNO experiment [6] could lead to a reduction only of the error in $\sin^2\theta_{12}$ to 21% [85,84]. If instead of 766.3 Ty one uses simulated 3 kTy KL data in the same data analysis, the 3σ errors in Δm^2_{21} and $\sin^2\theta_{12}$ diminish to 7% and 18% [84].

The most precise measurement of Δm^2_{21} could be achieved [85] using SK doped with 0.1% of Gadolinium (SK-Gd) for detection of reactor $\bar{\nu}_e$ [86]: SK gets the same flux of reactor $\bar{\nu}_e$ as KamLAND and after 3 years of data-taking, Δm^2_{21} would be determined with a 3.5% error at 3σ [85]. A dedicated reactor $\bar{\nu}_e$ experiment with a baseline L ~60 km tuned to $\bar{\nu}_e$ survival probability minimum, could provide the most precise determination of $\sin^2\theta_{12}$ [87,88,84]: with statistics of ~60 GWkTy and systematic error of 2% (5%), $\sin^2\theta_{12}$ could be measured with uncertainty of 6% (9%) at 3σ [84]. A generic LowNu $\nu - e$ elastic scattering experiment, designed to measure the pp $\nu_\odot$-flux with an error of 3% (1%), would permit to determine $\sin^2\theta_{12}$ with an error of 14% (19%) at 3σ [84]. The inclusion of the uncertainty in θ_{13} ($\sin^2\theta_{13}$ <0.05) in the analyses increases the quoted errors by (1-3)% [84].

Measuring $|\Delta m^2_{\rm A}|$, $\theta_{\rm A} \equiv \theta_{23}$, $sign(\Delta m^2_{\rm A})$ and δ. The expected 3σ uncertainties in $|\Delta m^2_{\rm A}|$=$|\Delta m^2_{31}|$ from studies of ν_μ-oscillations in i) MINOS + CNGS [55,57,58], ii) NOνA [60] and iii) T2K (SK) [59] experiments, if the true $|\Delta m^2_{31}|$= $2{\times}10^{-3}$ eV^2 and true $\sin^2\theta_{23}$=0.5, read [79]: i) 26%, ii) 25% and iii) 12%. T2K (SK) and NOνA experiments will measure also $\sin^2 2\theta_{23}$ with a high precision - (1-2)% at 1σ. However, they would not be able to resolve the θ_{23}–$(\pi/2 - \theta_{23})$ ambiguity if $\sin^2 2\theta_{23}$ <1. T2K and NOνA are planned to begin in 2009 and 2011 (or 2009 [60]) and in their first phase, both experiments will use ν_μ-beams. The data from this phase will not allow to determine the $sign(\Delta m^2_{31})$. Even if $\sin^2 2\theta_{13}$ ~0.10, without the knowledge of $sign(\Delta m^2_{31})$ it would be impossible to get unambiguous information

on CP-violation in ν-oscillations (induced by δ) using only the phase-I T2K and NOνA data [79]. If $\sin^2 2\theta_{13} \gtrsim 0.05$, information on $sign(\Delta m^2_{31})$ and $\sin^2\theta_{23} \gtrsim 0.5$ might be obtained in $\nu_{\rm A}$-experiments by studying the Zenith angle dependence of the multi-GeV e- and μ- like, and/or $\mu^{\pm}$, events [89]. Resolving all parameter degeneracies [90,80] and determining whether δ takes a CPV value if $\sin^2 2\theta_{23}$ <1 and no information on $sign(\Delta m^2_{31})$ is available, would be a formidable task and would require high statistics (phase-II) data on ν_{μ}- and $\bar{\nu}_{\mu}$- oscillations both from T2K and NOνA and data on θ_{13} from a reactor experiment [79,90,80]; if $\sin^2 2\theta_{13} \lesssim 0.01$, data from SPL+$\beta$-beam experiments [61] or from experiments at ν-factory [62] might be required.

5. Instead of Conclusions

We are at the beginning of the road leading to a comprehensive understanding of the patterns of neutrino masses and mixing and of their origin. The road is not easy and we do not quite know how long our "journey" will take, how difficult it will be and what we will finally discover. However, I am sure the "view" that will open to us from the "summit" at the end of this "journey" will be of dazzling clarity, perspective and beauty (Fig. 4).

Figure 4. Landscape by Y. Hiroshige (Ukiyoe master, Edo epoch, Japan).

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